An Automotive SMA Mirror Actuator: Modeling, Design, and Experimental Evaluation

ERIC WILLIAMS AND MOHAMMAD H. ELAHINIA*

Dynamic and Smart Systems Laboratory, Mechanical Industrial and Manufacturing Engineering Department, University of Toledo, Toledo, OH 43606, USA

ABSTRACT: Shape memory alloys (SMAs) provide compact and effective actuation for a variety of mechanical systems. Automotive applications of these materials, although very attractive, have not been fully explored. The lack of activities in this area is partly due to the complex thermomechanical behavior of these materials. In this work a two degree of freedom SMA actuator has been developed to orient the position of a rearview mirror for automotive applications. This paper presents the design, modeling, and experimental evaluation of this SMA mirror actuator. To evaluate the performance of the actuator a prototype SMA actuated mirror is designed and fabricated. Experimental results are presented to demonstrate the effectiveness of the SMA actuator in positioning the mirror.

Key Words: actuator, shape memory, automotive.

INTRODUCTION

SHAPE memory alloys (SMAs) are a group of smart materials that can effectively change their shape and provide actuation by restoring their memorized geometry. The reversible mechanism behind shape memory alloy actuation is a solid-state phase transformation that takes place in response to variation of temperature and stress. The distinct thermomechanical behavior of SMAs is the result of a transformation from the austenite (parent) phase to martensite (product) phase and vice versa (Brinson, 1993a). These alloys have very high energy density; therefore, actuators that implement these alloys are compact and lightweight alternatives to other types of actuators such as DC motors and solenoids. SMA actuators are an effective way to reduce weight and to minimize the complexity of various systems.

Among different models developed for the thermomechanical behavior of SMAs, phenomenological models have been most widely used for actuator design and control. A phenomenological model comprises two main parts: a constitutive model that defines the thermomechanical behavior of the material by expressing the stress as a function of temperature, strain, and martensite fraction; and a phase transformation kinetics model describes the martensite fraction as a function of temperature and stress of the material.

More details on the thermomechanical modeling of SMAs can be found in (Liang, 1990; Brinson, 1993b; Elahinia, 2004).

Most of the SMA actuators utilize these alloys in the wire form. The SMA actuators are designed to use the shape memory effect in creating motion and force. When an SMA wire is heated, the material applies a large amount of force and displacement while returning to a memorized length. Joule heating is an effective and simple way for actuating SMA components. Existing applications of SMA actuators include devices such as active endoscopes, rifle stabilizer systems, commercial linear actuators, helicopter rotor actuators (Ikuta et al., 1998; Chopra, 2002; McGregor, 2003; Barnes et al., 2005).

Automobiles have an increasing need for actuators in response to the increasing number of luxury features available as well as the growing demand of drive by wire technology. Actuators that are commonly found in automobiles are mostly based on DC motors or solenoids. DC motors require multiple gears and pulleys to translate their fast rotary motion into slow linear actuation. Solenoid actuators require wire wrapping and magnetic materials. SMAs can potentially simplify and reduce the cost of many automotive actuators (Marshall et al., 1999; Jones et al., 2002). This paper presents the design and evaluation of an automotive mirror actuator based on SMA wires. The mechanical and electronic design process of the actuator are explained in detail and the results of the modeling and experimental evaluation of the mirror are presented.

*Author to whom correspondence should be addressed.
E-mail: mohammad.elahinia@utoledo.edu
Figures 1, 2, 4, 5 and 8–12 appear in color online: http://jim.sagepub.com
SMA MIRROR ACTUATOR DESIGN

In an automotive external rearview mirror, the actuator typically consists of two DC motors and a gear assembly, as shown in Figure 1. A SMA actuator for mirror positioning, on the other hand, can simply be composed of joints and fixtures that can drastically reduce the number of necessary components and simplify the required assembly. The main objectives in developing the SMA mirror actuator are to reduce the number of moving parts and to reduce the manufacturing cost of the mirror. The design concept for the actuator is depicted in Figure 2, through a simple mechanism four SMA wires rotate the mirror about its two axes. As shown in Figure 2 the SMA actuated mirror is much simpler than the traditional actuator. The two DC motors of the conventional actuator is replaced by four lengths of SMA wires. When one of these wires is heated through the resistive heating, the wire contracts due to phase transformation. By adjusting the electric voltage, it is possible to achieve controlled temperature in the SMA wire. By controlling the temperature, the level of actuation is controlled. With the proper selection of a control algorithm, therefore, it is possible to position the mirror to a desired orientation. As mentioned before, the orientation of the mirror is defined by its rotation about two perpendicular axes. In order to identify the design objectives of the mirror, a standard powered side mirror was analyzed. This side mirror had a range of $\pm 9^\circ$ for each of the two angles of rotation. One of the design objectives for the SMA mirror was that this range should be met. The mirror should also have the capability to be positioned quickly and maintain stability throughout operation and rest similar to that of current mirrors with minimal sensitivity to variation in ambient temperature.

SMA wires provide the actuation force and displacement for the mirror. For properly selecting the SMA wire, the required force in the wire must be calculated. The amount of force is proportional to the diameter of the wire; larger diameter wires can be implemented to generate larger forces as needed. To increase the actuation life of the wire, it is essential to limit the strain of the SMA actuator to 4%. This limitation, along with the required displacement of the actuator, defines the length of the wire. When considering strain as a design factor, one must determine the initial prestretched length before the actuation strain is imposed. Subsequent strain of the wire will lead to stress and should be quantified kinematically to determine the strain of an individual wire:

$$\varepsilon_i = \frac{L(\theta_x, \theta_y, \theta_z) - L_a}{L_a},$$

where $\varepsilon_i$ is the strain in an arbitrary wire, $L(\theta_x, \theta_y, \theta_z)$ is the length of the wire as a function of the orientation angles, referenced from a coordinate system collocated at the center of the joint and $L_a$ is the minimum length of the wire when it is in the austenite phase. The length of each wire changes very little due to the change in $\theta_z$ because the rotation about this axis is very small and considered negligible. It should also be noted that the strain of each wire at the origin ($\theta_x = \theta_y = \theta_z = 0$)
is nonzero. The effective torque of the SMA actuator can be written as:

\[ \tau_{\text{wire}} = \vec{r}_{\text{wire}} \times \vec{F}_{\text{wire}}. \]  

(2)

The proposed mirror is of the rotary type, meaning that the input to generate motion is torque as opposed to force. A relationship for the torque delivered by the wire about the joint can be found in (2), where \( \vec{r}_{\text{wire}} \) is the moment arm of the wire and \( \vec{F}_{\text{wire}} \) is the wire force. A simple representation of the kinematics of the mirror is shown in Figure 3. Since the unit vectors of each wire change with rotation of the mirror, the model updates the change in torque due to rotation in a vector representation.

It has been shown that creep can be a significant design factor for SMA actuators as referenced in Barnes et al. (2005). To account for creep in the design of the mirror, the initial strain in the wire should exceed the strain required for actuation by 20% [double the reported creep strain maximum (Barnes et al., 2005)].

The main design goals for the SMA actuator was for it to keep wire length and complexity to a minimum to avoid added costs. In order to reduce the wire length requirement, it is necessary to reduce the moment arm (2). Since this introduces a constraint on the moment arm, one must calculate the effective torques acting about the joint and allow for higher force requirements:

\[ \sum \tau = \tau_{\text{friction}} + \tau_{\text{gravity}} + \tau_{\text{wire}}. \]  

(3)

Given the low efficiency of SMA actuators and the limited power that a car battery can supply, it is necessary for the mirror actuator to achieve stable positioning without constant control input. To this end, similar to an approach taken by Singh et al. (2003) a passive friction brake was designed to hold the mirror in position after the control system has reoriented it. For this brake system, as shown in Figure 4, the upper limit of static friction should be high enough to hold the mirror in place against gravity and other disturbances so that the control system does not have to operate continuously. However, this value of frictional torque should not exceed the available wire torque in the direction of desired rotation. The friction characteristics of the joint for the initial prototype were tuned by a spring loaded cup. The displacement of the spring controlled the normal force on the cup thus changing the friction parameters.

### Electronic Design

The mirror actuator developed in this work accomplishes reorientation of the mirror via two pairs of antagonistic wires that are controlled by a switching circuit. In order to switch from supplying one wire with current to another wire, transistors are utilized that can be controlled by a Boolean control signal (on or off).

### MODELING

In order to effectively evaluate the conceptualized mirror actuator prior to building a prototype, a mathematical model was developed that generates qualitative results of the mirror actuator in various loadings. The model input is the electrical current applied to each SMA wire and its output is the orientation of the mirror. The state variables of the system are the stress of each SMA wire \( \sigma \), the temperature of each wire \( T \), the strain of each wire \( e \), and the phase state of each wire \( \xi = \xi_r + \xi_s \). As shown in Figure 5, the behavior of the

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**Figure 3. Kinematics of the SMA mirror actuator: two antagonistic wires rotate the mirror about each axis.**

**Figure 4. Spherical joint designed to provide the passive braking force for stabilizing the SMA actuated side mirror when the wires are not activated.**
SMA actuator is modeled by incorporating the following sub-models:

- the heat transfer of the wires,
- the constitutive characteristics of the wires,
- the phase transformation kinetics of the wires, and
- the dynamics and kinematics of the mirror.

The interdependence of these four sub-models define the nonlinear behavior of the SMA actuator. In this section, the main steps in developing the model are explained.

**Thermomechanical Behavior of SMA Wires**

The unique actuation behavior of SMAs is attributed to their distinct thermomechanical behavior. This behavior can be modeled by phenomenological models, which are more suitable for engineering applications (Elahinia and Ahmadian, 2005a). These models describe the qualitative behavior of shape memory materials and they consist of two parts: a constitutive model and a phase transformation (kinetics) model. These two parts of the model describe the behavior of each of the four SMA wire actuators (Table 1).

**CONSTITUTIVE MODEL**

Based on the work of Tanaka, Liang, and Brinson (Tanaka, 1986; Liang and Rogers, 1990; Brinson, 1993a), the thermomechanical behavior of SMAs can be described in terms of strain, $\varepsilon$, martensite fraction, $\xi$, and temperature, $T$. In the most general form, the thermomechanical constitutive equation is:

$$d\sigma = D(\varepsilon, \xi, T)d\varepsilon + \Omega(\varepsilon, \xi, T)d\xi + \Theta(\varepsilon, \xi, T)dT,$$

where $D(\varepsilon, \xi, T)$ is representative of the modulus of the SMA material, $\Omega(\varepsilon, \xi, T)$ is the transformational tensor, and $\Theta(\varepsilon, \xi, T)$ is related to the thermal coefficient of expansion.

**PHASE TRANSFORMATION KINETICS**

Phase transformation of SMAs from one solid state to another solid state is the mechanism by which the shape memory effect takes place. This transformation is from martensite (low-temperature phase) to austenite (high-temperature phase) and the reverse transformation. SMA phase transformation is achieved by raising and lowering the stress and temperature. As illustrated in Figure 6, there are four temperatures that define the austenite to martensite phase transformation. These four temperatures, which are called austenite start, austenite final, martensite start, and martensite final are shown, respectively, as $A_s$, $A_f$, $M_s$, and $M_f$. When the temperature of an SMA wire is raised beyond the austenite final temperature, the material will be 100% in austenite phase. Similarly, when the temperature decreases below the martensite final temperature, the alloy will be 100% in martensite phase. For any temperature between the extrema, both austenite and martensite coexist in the shape memory alloy. One of the

![Figure 5. Model elements of an SMA actuator.](image-url)
complexities in the behavior of these materials is shown in Figure 6: the four transformation temperatures are stress dependent. Thus for higher levels of stress, the transformation temperatures are higher. In other words, the hysteresis loop shifts to higher temperatures as the stress increases.

There are two classes of SMA actuators. One-way actuators (bias type) are composed of an SMA element and a bias spring. Two-way actuators (differential type) are composed of two SMA elements (Elahinia and Ashrafiuon, 2002). The differential type actuators have the advantage of easier and faster control while the bias type actuators require less power, making them more desirable for internally powered devices. Bias type actuators can be very slow due to the required cooling period for reverse actuation. The SMA mirror is based on a differential type actuator. The phase transformation kinetics model of a bias type actuator is different from the transformation in a differential type actuator. For both actuator types, it is important that the phase transformation model captures the effects of simultaneously varying stress and temperature (Elahinia and Ahmadian, 2005a, b).

In a bias type actuator, the SMA wire transforms from martensite to austenite when activated. It is sufficient to track only the overall martensite fraction for bias type actuators and disregard the components of twinned and detwinned martensite because of the reverse transformation from austenite to martensite and the heightened level of stress in the wire when cooling occurs. In reality there is a component of both twinned and detwinned martensite is present through this transformation. However, it is felt that the net result and effect of this detwinning is of minimal impact to bias type actuators.

For a differential type actuator, on the other hand, the bias force, which is provided by the opposing SMA wire, has a variable magnitude. As a result, the previously active SMA wire can transform to a combination of twinned and detwinned martensite variants upon cooling. The percentage of each variant of martensite depends on the stress level in the antagonistic SMA wire. The following sections describe the differences in the phase transformation kinetics of these two types of SMA actuators.

In order to include the possibility of transformation from austenite to a combination of twinned and detwinned martensite, Brinson introduced two internal variables (Brinson, 1993a):

$$\xi = \xi_T + \xi_S,$$

where $\xi_T$ and $\xi_S$ define the percentage of the twinned and detwinned martensite fractions, respectively. In the case of the SMA mirror actuator there are three possible phase transformations as illustrated in Figure 7.

The first type of phase transformation is when one wire is activated to cause rotation about one axis, if conditions

$$\dot{T} - \frac{\sigma}{C_A} > 0,$$

and

$$A_s + \frac{\sigma}{C_A} < T < A_f + \frac{\sigma}{C_A},$$

are satisfied the active wire undergoes martensite to austenite phase transformation:

$$\dot{\xi} = \frac{\xi_{MS}}{\xi_M} \{\cos[a_A(T - A_s) + b_A\sigma] + 1\}$$

$$\dot{\xi}_S = \frac{\xi_{MS}}{\xi_M} \xi,$$

$$\dot{\xi}_T = \frac{\xi_{MT}}{\xi_M} \dot{\xi},$$

where $\xi_{MS}$ and $\xi_{MT}$ are the detwinned and twinned martensitic fractions prior to the current transformation, respectively.
The second type of phase transformation, as shown in Figure 7, is when the active wire cools after actuation with the antagonistic wire remaining passive. If the conditions

\[
\frac{T}{C_0} < 0 \quad \text{and} \quad M_f + \frac{\sigma}{C_M} > T > M_s + \frac{\sigma}{C_M},
\]

are satisfied, for this case \(\xi_S\) remains unchanged and \(\xi_T\) increases:

\[
\xi_T = \frac{1 - \xi_{AT}}{2} \cos\left[\alpha M(T - M_f) + \beta M \sigma\right] + \frac{1 + \xi_{AT}}{2},
\]

\[
\dot{\xi}_S = 0,
\]

where \(\xi_{AT}\) is the twinned martensitic fraction prior to the current transformation.

The third case of phase transformation, as displayed in Figure 7, occurs when the opposing SMA wire is activated, the previously active wire remains passive and it will be extended at a low temperature \((< M_f)\). In this condition, \(\xi_S\) increases and \(\xi_T\) decreases:

\[
\xi_S = \frac{1 - \xi_{AS}}{2} \cos\left(\frac{\pi}{\sigma_s - \sigma_f} [\sigma - \sigma_f - C_M(T - M_s)]\right) + \frac{1 + \xi_{AS}}{2},
\]

\[
\dot{\xi}_T = -\dot{\xi}_S
\]

Dynamics and Kinematics of the Mirror

As shown in Figure 2 the four segments of wire are arranged to work in an antagonistic way about the two axes of rotation. Each of the four wires can be heated individually to generate a rotation about one of the two axes. In addition to the actuation force of each SMA wire, two resistive forces are applied to the mirror due to gravity and friction. The force of each SMA wire is proportional to the stress of the wire:

\[
\vec{F}_i = \sigma_i \times A \times \vec{u}_i
\]

where \(\sigma_i\), \(A\), and \(\vec{u}_i\), are the stress, the cross sectional area, and the unit vector along the direction of wire \(i\), respectively. Stress of each of the four wires is calculated separately with the provided constitutive model 4. The torque induced by gravity is very important to the actuator performance and without friction; a constant force is needed to counteract this torque. In the absence of an opposing force, the mirror will operate with a propensity for downward rotation due to gravity. As a result, the maximum achievable angle for the downward rotation will be larger than that of the upward rotation. A passive break system, as shown in Figure 4, provides the friction force so that stable positioning can be achieved without constant supply of power to the SMA wires. It is therefore important to incorporate the friction in the dynamic model of the mirror.

More specifically, using a friction model that best describes the physics of contact and reduces the frequency of switching between equations is essential in avoiding high frequency switching between differential equations, which can be hard to solve numerically. It is well known that classical friction models, such as Coulomb and Karnopp in which the relation between friction forces and the relative velocity of the contact surfaces are discontinuous, generate discontinuity in the model. This could cause a difficulty in integrating the equations of motion. In this work, after considering different types of friction laws, the Elasto-Plastic law that renders both presliding and stiction is selected (Dupont et al., 2000). Stiction is defined as the required displacement that must be exceeded before dynamic friction can be observed. This effect is modeled by a force that acts similar
to the elastic region of a typical stress–strain curve. This breakaway displacement is similar to the strain experienced in a tensile test when the test specimen begins to yield. In order to remove the discontinuity of the contact equation in the Elasto-Plastic model, parameter \( z \) is used to present the state of strain in frictional contact. The difference between this model and other models such as LuGre model is an extra parameter, which provides the stiction force and includes the presliding phenomenon. Thus, the Elasto-Plastic model is more comprehensive. Using this model, the equation of friction may be written as:

\[
f_f = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 v, \quad \sigma_i > 0,
\]

where \( \sigma_0 \) and \( \sigma_2 \) are Coulomb and viscous friction parameters and \( \sigma_1 \) provides damping for the tangential compliance. Displacement \( z \) is defined by the deflection of an elastic member so that presliding displacement can be modeled and \( x \) is defined as the relative motion of the two bodies. The function that governs the derivative of this bristle-like deflection can be written as (Dupont et al., 2000):

\[
\dot{z} = \dot{x}(1 - \alpha(z, \dot{x}) \frac{\sigma_0}{f_s(x)} \text{sgn}(\dot{x}) z), \quad \frac{\sigma_0}{f_s(x)} > 0, \quad i \in \mathbb{Z}.
\]

Also another function, \( \alpha(x, z) \), is used to incorporate the stiction effect in the model. Requirements for this function are discussed in Dupont et al. (2000). In this work, this function is defined as:

\[
\alpha(z, \dot{x}) = \begin{cases} 
0 & \text{if } |z| < z_{ba} \\
\frac{1}{2} \sin \left( \pi \frac{z - (z_{max} + z_{ba})/2}{z_{max} - z_{ba}} \right) + \frac{1}{2} & \text{if } z_{ba} \leq |z| \leq z_{max} \\
1 & \text{if } |z| \geq z_{max}
\end{cases}
\]

\( z_{ba} \) is the breakaway displacement and is defined such that the model behaves elastically for \( |z| < z_{ba} \).

**Heat Transfer Model**

The assumed SMA wire heat transfer equation consists of electrical heating and natural convection:

\[
m C_p \frac{dT}{dt} = \frac{V^2}{R} - h(T) A_c (T - T_\infty) - m \Delta H \dot{\xi},
\]

where \( m \) is mass per unit length, \( C_p \) is the specific heat, \( R \) is electrical resistance of the wire, \( h(T) \) is the heat convection coefficient, \( A_c \) is the circumferential area of the SMA wire. Also, \( V \) is the applied voltage, \( T \) is the ambient temperature, \( \Delta H \) is the latent heat, and \( \dot{\xi} \) is the phase transformation rate. The orientation of the wire when heated also has an effect on the convection heat transfer coefficient (Szykowny and Elahinia, 2006).

**RESULTS**

In this section, simulation results based on the model are presented as well as the experimental results obtained from the system. For simulation purposes, the model is implemented in MATLAB/Simulink/SimMechanics.

The first simulation test involving the robustness of the mirror is conducted by sweeping the ambient temperature through a range of temperatures that are possible exposures for an automobile (−20 to 50°C). Results from this first test can be found in Figure 8. As suspected, the output of the actuator varies as does the temperature when the applied voltage is constant. As the temperature decreases, it can be seen that the output is reduced. Also notable is an increase in the amount of time it takes for the alloy to begin actuating. Therefore, it is important that the control logic, which regulates the voltage to each SMA wire, considers the effect of ambient temperature.

Phase transformation in the antagonistic SMA actuator takes place due to temperature and/or stress variation. Figure 9 illustrates the thermomechanical loading of one of the SMA wires in the mirror actuator. Initially, the wire is at ambient temperature and stress in the wire is zero. This condition simulates the state of the SMA actuated mirror when it is in the middle position and each of the SMA wires at room temperature are made of two martensitic variants: half twinned martensite and half detwinned martensite [\( \xi_T = (1/2) \), \( \xi_S = (1/2) \), and \( \xi = 1 \)]. Consequently, the wire is heated beyond the austenite start (\( A_s \)) and through the austenite final (\( A_f \)) temperatures. As a result, both...
martensitic variants are transformed to austenite ($\xi_T = 0$, $\xi_S = 0$, and $\xi = 0$). At the end of actuation, when the wire is cooled below the martensite start ($M_s$) and through the martensite final ($M_f$) temperatures, the wire becomes fully twinned martensitic ($\xi_T = 1$, $\xi_S = 0$, and $\xi = 1$). Finally, when the opposing SMA wire is heated the stress of the SMA wire is increased and the wire becomes detwinned martensite ($\xi_T = 0$, $\xi_S = 1$, $\xi = 1$).

To evaluate the performance of the SMA mirror actuator, a prototype mirror was built. The mirror was made of a spherical joint and four SMA wires. The Flexinol SMA wires had a diameter of 200 $\mu$m and were supplied by Dynalloy Inc. Three 50 $\mu$m resolution laser displacement sensors were used in order to obtain the three points needed to detect the mirror plane angle. To this end, the three laser displacement sensors were fixed within a box that held them parallel. A picture depicting the design for this experimental setup is shown in Figure 10. Each of the three measurements had its particular $x$ and $y$ coordinates determined by the distance between each beam. The value determined by the sensor was then taken as the $z$ coordinate and a unit vector was defined by the three points coinciding with each laser measurement. Additionally, a dSPACE 1104 controller was used along with an Agilent 6543A power supply for conducting the experiments.

Figure 11 shows the results of an experiment to evaluate the SMA mirror. In this experiment, one pair of antagonistic SMA wires is actuated in sequence. The mirror rotates in each of the opposing directions with the desired angle. Ideally, the two degrees of freedom of the SMA actuator should be independent of each other. Rotation of the mirror about one axis, however, causes the wires to change in length and thus a coupling of the two degrees of freedom exists. This effect is illustrated in Figure 11. In this experiment, the horizontal axis has $\sim 2^\circ$ of drift when the mirror is actuated about the vertical axis.

Figure 12 illustrates the effect of current magnitude on rotation of the SMA actuated mirror. In these experiments, the opposing SMA wires are
activated in sequence to rotate the mirror about its vertical axis. In order to keep the conditions consistent for each test, all of the tests were conducted within 30 min at room temperature. This experiment illustrates the possibility of controlling the rotation through regulating the electric current applied to the proper SMA wire.

CONCLUSION

A concept SMA automotive actuator was evaluated. Four SMA wires were used to develop an antagonistic type SMA actuator to orient an automotive side mirror. The main benefits of the new actuator is in simplifying the design and in providing cost savings. The mirror actuator presented in this work demonstrated the ability of a simple configuration of SMA wires to replace a more complex arrangement of motors, gears, and joints. A comprehensive model was developed for the actuator that enabled design evaluation and will be used for future control system design. This model includes all the major subsystems of the mirror: dynamics and kinematics of the mirror, thermomechanical material behavior of the SMA wires, and heat transfer model of the wire. Experimental evaluations as well as simulations show the effectiveness of the new SMA actuator. In the future, the control problem of the mirror will be investigated in order to overcome the sensitivity of the actuator to the ambient temperature variation.
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