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Position Control of a Three-link Shape Memory Alloy Actuated Robot

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ABSTRACT: A simple and robust position controller is proposed for a small planar three-degree-of-freedom robot arm actuated by two shape memory alloy (SMA) actuators and a servomotor. A simple model of the robot is used for controller development. The model combines robot kinematics and dynamics with crude models of SMA wire heat convection, constitutive law, and phase transformation. It is then used to estimate ‘optimal’ parameters for the position controllers. The controllers are based on variable structure control approach and their development is of an evolutionary nature starting with a simple switching surface of position tracking errors and followed by the addition of velocity and integral tracking errors, respectively. The final controller is shown to be particularly fast since it heats the SMA wires with maximum available voltage, but avoids overshoot thereby avoiding the slow natural cooling process compared. Several experiments have been performed with a desktop prototype of the robot. The experimental results verify the effective and robust performance of the controllers despite significant modeling inaccuracies during the controller parameter development process. An additional advantage of the controller is that it can be implemented on a controller board with very limited computation capacity.

Key Words: SMA actuator, variable structure control, robot manipulator.

INTRODUCTION

SHAPE memory alloy (SMA) miniature actuators have received significant attention in recent years because of their high power to mass ratio, frictionless actuation, silence, and simplicity of their mechanisms. Despite these special features, SMAs have not been fully exploited by developers, particularly in devices employing feedback control systems. These devices are inherently nonlinear, characterized by a hysteresis loop and saturation. Although numerous analytical and phenomenological models have been developed for SMAs over the last forty years, it is difficult to choose proper models for different applications. Furthermore, a given model may contain many unknown parameters, discouraging from further attempts to design a model-based control system. Generally, the SMA actuator weak points are energy efficiency and control difficulties.

There are two classes of SMA actuators. The bias-type actuators are one-way actuators composed of an SMA element and a bias spring. The differential actuators are two-way actuators made of two SMA elements (Hashimoto et al., 1985; Kuribayashi, 1986; Gharaybeh and Burdea, 1995). The differential-type actuators have the advantage of easier and faster control while the bias-type actuators (Gorbet and Russel, 1995) require less power, making them more desirable for low-power applications. The bias-type actuators can be very slow if there is an overshoot thus requiring cooling and the work of the bias spring for reverse actuation. The robotic applications of SMA actuators include articulated hand and rotary joint motion (Hashimoto et al., 1985; Kuribayashi, 1986).

Considerable research for modeling the microscopic and macroscopic behavior of SMAs has been performed in the past forty years. Since the mechanical behavior of SMAs is closely related to microscopic phase transformation (between martensite and austenite phases), constitutive relations are not merely stress-strain based and must include phase transformation and heat transfer characteristics (Kuribayashi, 1986; Liang, 1990; Tanaka, 1986; Ikuta et al., 1991; Arai et al., 1994; Kafka, 1994; Shahin et al., 1994). In recent years, Preisach modeling of the hysteretic behavior of the SMA has

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helped to predict the SMA actuator response more precisely (Choi et al., 2004).

In the controls area, researchers have explored linear controllers (Ikuta et al., 1991; Reynaerts and Van Brussel, 1991; Arai et al., 1994; Madill and Wang, 1994; Carpenter et al., 1995; Van der Wijst et al., 1997) as well as pulse width modulation (Hashimoto et al., 1985; Kuribayashi, 1986; Tanaka and Yamada, 1991; Gharaybeh and Burdea, 1995; Gorbet and Russel, 1995). Nonlinear control schemes include fuzzy logic (Nakazato et al., 1993), feedback linearization (Arai et al., 1994), optimal control (Lee and Mavroidis, 2002), and variable structure control (Choi and Cheong, 1996; Grant and Heyward, 1997; Elahinia and Ashrafiuon, 2002; Song et al., 2003; Elahinia et al., 2004a, b). Among these, the variable structure controllers (VSC) have shown to be the most effective in the presence of modeling uncertainties and disturbances.

A small-scale three-degree-of-freedom robot has been designed with two bias SMA actuators and a servomotor. The robot is a preliminary prototype to show the potential of SMA actuators in designing microscale robots. The robot is designed with as little weight as possible such that it can be installed in a small mobile robot without causing it to turn over for its full range of motion. In this study, we develop and experimentally verify a nonlinear position controller for the robot, which can be implemented on any small on-board controller card with limited computation capacity. A system model is developed based on the robot dynamics and simple SMA constitutive, heat convection, and macroscopic phase transformation characteristics. The model is used to propose ‘optimal’ parameters for position controllers based on VSC due to significant model uncertainty and nonlinear behavior of the SMA actuators. The switching surface of the VSC is initially designed based on position tracking error only. Velocity and integral tracking errors are then numerically added to achieve sliding modes and reduce the steady-state errors. Since our SMA actuators are of bias type, control parameters are tuned to avoid overshoot while still reaching the desired position quickly. Results obtained from simulations and experiments indicate that the proposed controller can effectively and accurately control the three-link arm.

The SMA Robot Prototype and Model

The SMA manipulator is a planar three-degree-of-freedom robotic arm with three revolute joints as shown in Figure 1. A servomotor drives the first link while the SMA actuators rotate links 2 and 3. There is also an additional servomotor operating the gripper. The SMA actuation mechanism consists of two collocated SMA wires, a bias spring, and a pulley that provides the rotational movement of the joints 2 and 3, as shown in Figure 2. The SMA wires are initially stretched to about 4% strain. Electrical heating of the wires will contract and provide one-way CCW rotation through the pulleys. Since the actuators are of bias type, torsional springs are designed to stretch back the SMA wires and provide CW rotation during cooling. Note that such actuators enjoy better efficiency compared to differential SMA actuators (Thrasher et al., 1994).

Figure 1. The three-link SMA actuated robot carrying a load.
Table 1 summarizes the joint and link properties of the robot. The robot is capable of lifting at least 100 g.

The model of the SMA robot consists of robot kinematics and dynamics, constitutive model, and convection heat transfer between the wire and the surrounding air. Note that we are not modeling latent heat effects. The model along with some open-loop experiments is used to estimate the 'optimal' control parameters since the controller is not model-based.

**Kinematics and Dynamics**

The inverse kinematic solution of any three-link planar robot with link lengths \((l_1, l_2, l_3)\) based on any desired end-effector Cartesian position and orientation \((x_d, y_d, \theta_d)\) is straightforward,

\[
\begin{align*}
\theta_2 &= -\cos^{-1}\left(\frac{r_x^2 + r_y^2 - (l_1^2 + l_2^2)}{2l_1l_2}\right); \\
\theta_1 &= \cos^{-1}\left(\frac{r_xl_2\sin\theta_2 + r_y(l_1 + l_2\cos\theta_2)}{r_x^2 + r_y^2}\right) \\
\theta_3 &= \theta_d - (\theta_1 + \theta_2)
\end{align*}
\]

where \(r_x = x_d - l_3 \cos \theta_d\) and \(r_y = y_d - l_3 \sin \theta_d\).

The standard equations of motion for a three-link planar robot with link lengths \((l_1, l_2, l_3)\) based on any desired end-effector Cartesian position and orientation \((x_d, y_d, \theta_d)\) is straightforward.

\[
m_i \ddot{x}_i + m_i \ddot{y}_i = \tau_i - \tau_{gi} - \tau_{ui} + \tau_j - m_i \ddot{\theta}_i, \quad i = 2, 3
\]

For each link \(i\), \(m_i\) and \(m_{i\theta}\) are the components of the standard mass matrix of the three-link planar robot, \(\tau_c\) the torque due to centripetal accelerations, \(\tau_{gi}\) the moment due to gravity, \(\tau_{ui}\) the torque due to torsional spring and joint friction, and \(\tau_j\) the torque applied by the SMA actuator,

\[
\tau_j = 2r_p(\sigma_i A_i), \quad A_i = \pi r_i^2, \quad i = 2, 3
\]

where \(r_p\) is the pulley radius, \(\sigma_i\) the wire normal stress, \(A_i\) the wire cross-sectional area, and \(r_i\) the wire radius for link \(i\). The factor 2 is used because each link has a wire wrapped twice around the pulleys to provide larger wire contraction (motion) leading to larger joint motion range and workspace. The SMA wire strain rate \(\dot{\varepsilon}_i\) of link \(i\) is kinematically related to its joint velocity \(\dot{\theta}_i\) as:

\[
\dot{\varepsilon}_i = \frac{-2r_p \dot{\theta}_i}{l_{wi}}, \quad i = 2, 3
\]

where \(l_{wi}\) is the initial length of wire \(i\).

**SMA Wire Heat Transfer Model**

The SMA wire free convection heat transfer model is required to determine the rate of heating and cooling due to changes in electric current in the wire. The free convection heat transfer from the wire to the surrounding air is defined by (Shahin et al., 1994):

\[
m_{wi}c_p \frac{dT_i}{dt} = \frac{V_i^2}{R_i} - hA_i(T_i - T_\infty), \quad i = 2, 3
\]

where \(m_{wi}\) is the mass, \(V_i\) the applied voltage, \(c_p\) the specific heat, \(h\) the heat convection coefficient, \(T_i\) the temperature, \(A_i\) the surface area of wire \(i\), and \(T_\infty\) the ambient temperature. The SMA wire properties and the heat transfer model parameters used in this study are presented in Table 2. Note that the value of \(h\) was determined and averaged through several electric heating experiments and \(T_\infty\) was controlled at 20°C.

**SMA Constitutive Model**

To determine the induced normal stress in the SMA wire, the Liang (1990) model is used. In this model, the
rate of change in stress, \( \dot{\sigma} \), is simply a function of rate of change of strain, \( \dot{\varepsilon} \), temperature, \( T \), and the induced Martensite fraction, \( \xi_i \), in the wire:

\[
\dot{\sigma}_i = D \dot{\varepsilon}_i + T_i + \Omega_i \dot{\xi}_i, \quad i = 2, 3
\]

(6)

where \( D \) is the Young’s modulus, \( \theta_T \) is the thermal expansion factor, and \( \Omega_i \) is the phase transformation contribution factor of wire \( i \).

**SMA Phase Transformation Model**

The amount of heating and cooling determine the fully induced or the extent of the partially induced forward and reverse transformations. The martensite (volume) fraction for wire \( i \), \( 0 \leq \xi_i \leq 1 \), is a measure of the extent of these transformations. Reverse transformation from martensite to austenite during heating may be defined as (Liang, 1990):

\[
\xi_i = \frac{1}{2} \left\{ \cos[a_A (T_i - A_f) + b_A \sigma_A] + 1 \right\}
\]

if \( A'_f \leq T_i \leq A'_s \) and \( \dot{\xi}_i < 0, \quad i = 2, 3 \quad (7) \]

where \( a_A = \pi/(A_f - A_s) \), \( b_A = -(a_A/C_A) \), and \( C_A \) is a material constant indicating the effect of stress on the reverse transformation temperatures. Also, \( A_s \) and \( A_f \) are austenite phase transition start and final temperatures, while \( A'_s \) and \( A'_f \) are their stress modified values, \( A'_s = A_s + (\sigma_s/C_A) \) and \( A'_f = A_f + (\sigma_f/C_A) \).

The forward transformation equations from austenite to martensite during cooling can be represented by Brinson’s model (Brinson, 1993). However, this model is not necessary since one of the objectives of our set point controller is to avoid cooling altogether. The parameters used for Equations (5) and (6) are presented in Table 3.

**Model Validation**

We have performed several experiments to verify our model. The most appropriate comparison, however, is the open-loop joint angle responses of the robot to applied voltages of varying magnitudes. Figure 3 shows the open-loop response of Joint 2 to constant voltages ranging from 5 to 8.2 V. Figure 4 shows the experimental response of the same joint to constant applied voltages ranging from 5.5 to 7.4 V. It is clear that experiments and simulations have similar results with the bulk of actuation occurring in the small range

---

**Table 2. SMA wire and heat transfer model parameters (SI units).**

<table>
<thead>
<tr>
<th>Diameter</th>
<th>Length</th>
<th>Mass</th>
<th>Initial strain</th>
<th>( C_p )</th>
<th>( h )</th>
<th>( R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wire 2</td>
<td>200e-6</td>
<td>0.9448</td>
<td>2.02e-4</td>
<td>0.04</td>
<td>837.4</td>
<td>85</td>
</tr>
<tr>
<td>Wire 3</td>
<td>150e-6</td>
<td>0.8701</td>
<td>1.13e-4</td>
<td>0.04</td>
<td>837.4</td>
<td>85</td>
</tr>
</tbody>
</table>

**Table 3. SMA wire constitutive and phase transformation parameters (SI units).**

<table>
<thead>
<tr>
<th>( A_s )</th>
<th>( A_f )</th>
<th>( C_A )</th>
<th>( \theta_T )</th>
<th>( \Omega )</th>
<th>( D_M )</th>
<th>( D_A )</th>
<th>Initial stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wire 2</td>
<td>68</td>
<td>78</td>
<td>10.3e6</td>
<td>-0.055e6</td>
<td>-2.06e9</td>
<td>28e9</td>
<td>75e9</td>
</tr>
<tr>
<td>Wire 3</td>
<td>68</td>
<td>78</td>
<td>10.3e6</td>
<td>-0.055e6</td>
<td>-2.06e9</td>
<td>28e9</td>
<td>75e9</td>
</tr>
</tbody>
</table>

---

Figure 3. Simulation of open-loop joint 2 response to incremental constant voltages.

Figure 4. Open-loop experimental joint 2 response to incremental constant voltages.
of 6–7.4 V, emphasizing the complexity of the control problem.

**CONTROLLER DESIGN**

We define the state vector from the joint angles, angular velocities, SMA wire temperatures, wire stresses, and martensite fractions as:

\[
x = [\theta_2 \quad \theta_3 \quad \dot{\theta}_2 \quad \dot{\theta}_3 \quad T_2 \quad T_3 \quad \sigma_2 \quad \sigma_3 \quad \xi_2 \quad \xi_3]^T \tag{8}
\]

The state equations are derived from Equations (2), (5), (6), and the time derivative of Equation (7) as:

\[
\begin{align*}
\dot{x}_1 &= x_3 \\
\dot{x}_2 &= x_4 \\
m_{22}\dot{x}_3 + m_{23}\dot{x}_4 &= \tau_{c2}(x_1, x_2, x_3, x_4) \\
\quad - \tau_{g2}(x_1, x_2) - \tau_{c2}(x_1, x_2) + 2r_{p2}A_2x_7 - m_{12}\dot{\theta}_1 \\
m_{23}\dot{x}_3 + m_{33}\dot{x}_4 &= \tau_{c3}(x_1, x_2, x_3, x_4) \\
\quad - \tau_{g3}(x_1, x_2) - \tau_{c3}(x_1, x_2) + 2r_{p3}A_3x_8 - m_{13}\dot{\theta}_1 \\
m_{w2c2p}\dot{x}_5 &= -h_{A2}(x_5 - T_\infty) + V_2^2 / R_2 \\
m_{w3c2p}\dot{x}_6 &= -h_{A3}(x_6 - T_\infty) + V_3^2 / R_3 \\
\dot{x}_7 &= -\theta_1\dot{x}_3 - \Omega_2\dot{x}_0 = -\frac{r_{p2}}{l_{w2}} D x_3 \\
\dot{x}_8 &= -\theta_2\dot{x}_4 - \Omega_2\dot{x}_0 = -\frac{r_{p3}}{l_{w3}} D x_4 \\
\dot{x}_9 + S_{T2}(x_3, x_7)\dot{x}_7 + S_{e2}(x_3, x_7)\dot{x}_5 &= 0 \\
\dot{x}_{10} + S_{T3}(x_5, x_7)\dot{x}_8 + S_{e3}(x_5, x_7)\dot{x}_6 &= 0 \tag{9}
\end{align*}
\]

The coefficients \(S_{Tj}\) and \(S_{ej}\) are defined zero except during reverse transformation (heating) and if \((A_\xi + (x_i + 6/C_A)) < x_i < (A_\xi + (x_i + 6/C_A))\):

\[
\begin{align*}
S_{Tj} &= \frac{\xi_{Mi}}{2} \sin[a_\xi(x_i + 4 - A_\xi) + b_\xi x_i + 6]a_\xi, \quad i = 2, 3 \\
S_{ej} &= \frac{\xi_{Mi}}{2} \sin[a_\xi(x_i + 4 - A_\xi) + b_\xi x_i + 6]b_\xi, \quad i = 2, 3 \tag{10}
\end{align*}
\]

It is also possible to reduce the number of state variables by eliminating the martensite fractions from the equations since the constitutive model of Equation (6) represents an algebraic loop. However, this issue is not addressed here.

The model presented by the above nonlinear state equations is very crude and the parameters involved are not well known. Hence, we use VSC approach, which provides effective and robust means of controlling linear and nonlinear plants in the presence of such significant modeling uncertainties (Hung, 1993; Utkin, 1997). In their approach, a surface \(s\) is defined using the state tracking errors, which serves a switching surface from one control structure to another. A model-based control is then developed using the Lyapunov stability theory. However, we will only use the switching surface in this study since our control law is not based on the system model.

Since only joint angles, \(\theta_2\) and \(\theta_3\), are measured using position encoders, we initially define the surface \(s_i\) as a function of position error \(\dot{\theta}_j\),

\[
s_i = \lambda_P \ddot{\theta}_j, \quad \dot{\theta}_j = \theta_i - \theta_{di}, \quad i = 2, 3 \tag{11}
\]

where subscript \(d\) denotes the desired values and \(\lambda_P\) are the position gains. But such surfaces will not result in sliding modes since velocity errors are not used.

Next, joint angular velocities, \(\dot{\theta}_2\) and \(\dot{\theta}_3\), are numerically calculated from the position encoder data and their errors are appended to position errors to define proper sliding surfaces,

\[
s_i = \lambda_P \dot{\theta}_j + \lambda_V \ddot{\theta}_j, \quad \dot{\theta}_j = \theta_i - \theta_{di}, \quad i = 2, 3 \tag{12}
\]

where \(\lambda_V\) are the velocity gains. The velocity feedback will be available through numerical differentiation of position encoder data. Note that \(-\lambda_P/\lambda_V\) is the slope of the sliding surface \(s_i\) in the phase plane and dictates the speed of the system response.

Finally, integral tracking errors are added to the surface using the integral gain, \(\lambda_I\), to reduce steady-state errors,

\[
s_i = \lambda_P \dot{\theta}_j + \lambda_V \ddot{\theta}_j + \lambda_I \int_0^t \ddot{\theta}_j dt, \quad i = 2, 3 \tag{13}
\]

The structure of VSC is defined based on the hardware limitations and the behavior of the SMA actuators. We first apply the highest available voltage, \(V_{\text{high}}\), to achieve the fastest possible response. A boundary layer, \(\phi_i\), is defined for each surface \(s_i\). With the proper tuning of the controller gains, the voltages will be sufficiently reduced once these boundary layers are reached to avoid overshoot, which requires time-consuming cooling process and results in chattering. The reduction of the voltage is based on the difference between \(V_{\text{high}}\) and a lower limit voltage \(V_{\text{low}}\),

\[
u_i = \begin{cases} 
V_{\text{high}} & \text{if } \frac{s_i}{\phi_i} > +1 \\
\frac{s_i}{\phi_i} & \text{if } \frac{|s_i|}{\phi_i} < 1, \quad i = 2, 3 \\
V_{\text{low}} & \text{if } \frac{s_i}{\phi_i} < -1
\end{cases} \tag{14}
\]
The selection of boundary layer thickness, $\phi_i$, is presented in the following section. The stability of the above control system can be verified through sliding mode control theory. Since we have only one-way (bias type) actuators, the application of a constant high voltage to both SMA actuators will force the trajectories to reach the surfaces defined by Equations (12) or (13). All trajectories on the surfaces will slide to the origin (zero tracking error) since the surfaces are asymptotically stable by definition. Also note that within the boundary layer, the controller is identical to PID control law and the stability of the closed-loop system for the robot can be proved using Lyapunov stability theory as presented by Spong and Vidyasagar (1989) thus guaranteeing that the trajectory will remain on the surface.

**EXPERIMENTAL SETUP AND CONTROLLER PARAMETERS**

Two prototypes of the three-link SMA robot have been designed. The experimental desktop setup consists of a dSPACE board and panel, dSPACE software installed on a PC, two programmable power supplies, the robot, two position encoders (400–1200 pulses per revolution), two digital to analog converters for the encoders, and an EyeBot board to control the servomotor, as shown in Figure 5. The controller high and low voltages provided by the power supplies are $V_{\text{high}} = 20$ V and $V_{\text{low}} = 5$ V. Note that a low-pass filter is also used while taking the numerical derivative of the encoder position signal.

We tried boundary layer thickness values between 0 and $15^\circ$ for both $\phi_2$ and $\phi_3$ to study their effects and determine their appropriate values, as shown in Figure 6. This figure indicates that to avoid overshoot, the appropriate boundary layers for $\theta_2$ and $\theta_3$ should be about 15 and 8, respectively.

We have performed numerous iterative studies to determine the best range of values of position, velocity, and integral gains. However, we are only presenting the final results of our trials and in each case we use the median of the range of gains for the follow-up cases. Figure 7 indicates that since Joint 2 must carry larger loads, the position gain of Joint 2 must be higher than that of Joint 3, to minimize the steady-state error. These results suggest a range of 6–12 for position gain of Joint 2 and a range of 1–5 for position gain of Joint 3. The appropriate velocity gain for Joint 2 is in the range of 1–2 while for Joint 3 is in the range of 1–4, as shown in Figure 8. Figure 9 indicates that the integral gains must be relatively small. The ‘optimum’ integral gain for Joint 2 is in the range of 0.002–0.003 and for Joint 3 is between 0.03 and 0.035, resulting in the smallest steady-state error.

![Figure 5. Experimental setup or position control of the three-link SMA robot.](http://jim.sagepub.com)
Figure 6. Simulation of the effect of boundary layers on the controller performance: (a) joint 2 position and (b) joint 3 position.

Figure 7. Simulation of the effect of position gains on the controller performance: (a) joint 2 position and (b) joint 3 position.
Figure 8. Simulation of the effect of velocity gains on the controller performance: (a) joint 2 position and (b) joint 3 position.

Figure 9. Simulation of the effect of integral gains on the controller performance: (a) joint 2 position and (b) joint 3 position.
SIMULATION AND EXPERIMENTAL RESULTS

The three controllers introduced in Equations (11)–(14) were simulated for several tasks. The simulation result for a typical task where the end-effector is commanded to reach a point in its workspace with horizontal orientation is shown in Figure 10. It can be seen that the addition of velocity tracking error to the surface results in superior performance, while the addition of integral tracking errors further improves the steady-state results. Moreover, Figure 11 indicates that sliding modes are achieved for both the controllers when velocity errors are added to the surface.

Experiments with the controllers confirm the controller gain selection process through simulations. Figure 12 shows the experimental application of VSC with the position feedback, which results in a significant steady-state error and a slow response due to overshoot. The addition of velocity tracking errors to the surface in the experiment eliminates the overshoot and thus is very fast, as shown in Figure 13. It also reduces the steady-state error but the error remains considerable in terms of precision control. Figure 14 shows that the addition of integral tracking errors significantly reduces the steady-state error in the experiment. Figure 15 indicates that the controller is also able to reject disturbance, which is introduced in the form of a sudden front of cool air for a short duration.

CONCLUSIONS

A model-independent position control algorithm has been developed for a three-degree-of-freedom planar robot actuated by two SMA actuators and a servomotor. The nonlinear behavior of the SMA and its one-way actuation nature complicated the control problem since overshoot should be avoided altogether and because full state feedback was not available. In fact, only joint position feedbacks are available through encoders. A model consisting of robot kinematics and dynamics as well as simple SMA wire heat convection, constitutive law, and phase transformation equations was presented. The model was used to estimate ‘optimal’ parameters for the position controllers based on the variable structure control theory. The first controller was based on a switching surface composed position tracking errors since joint angles were the only measured states. The controller was improved by online numerical calculation of the angular velocities, which eliminated overshoot and reduced steady-state error. Finally, integral tracking errors were added to further reduce steady-state feedback. Experimental and simulation results verified the effective performance of the controller considering significant modeling uncertainties, inaccuracies in the model, and disturbances.
Figure 11. Achievement of sliding modes in both phase plots through simulation: (a) joint 2 phase plot and (b) joint 3 phase plot.

Figure 12. Experimental performance of VSC with only position feedback: (a) joint 2 position and (b) joint 3 position.

Figure 13. Experimental performance of VSC with position and velocity error feedbacks: (a) joint 2 position and (b) joint 3 position.
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